

Sonifying the Tides

| Remington Furman



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Sonifying the Tides

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Inspiration

About Me

- I'm Remington Furman
- I like using computers to compute
- Embedded Systems Engineer
- I've always lived near the Puget Sound in Washington State
- Not an oceanographer or tide expert

Inspiration: A Pacific Science Center Exhibit

- In 2016 I worked as an Exhibit Technician at the Pacific Science Center in Seattle, WA.
- I became very curious about this Puget Sound Model exhibit.
- It's a scale model used to simulate tidal currents in the Puget Sound.
- One of two built at the University of Washington Oceanography Department in the 1950s.



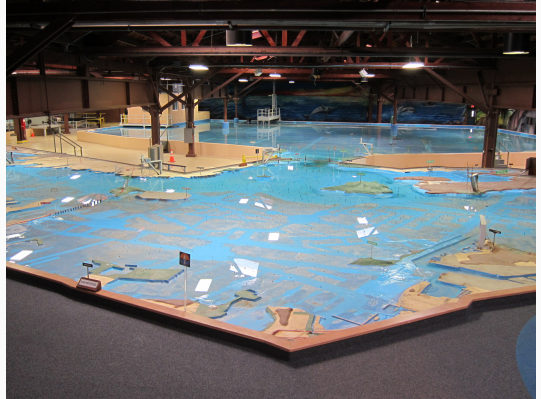
Inspiration: A Pacific Science Center Exhibit

- A large mechanical computer predicts the tides.
- A plunger moves up and down in the water to force the tides.
- A vacuum tube circuit controls the servo motor for the plunger.
- PacSci removed this exhibit from their floor this year. It was moved back to the UW Oceanography Department in October 2025.



The U.S. Army Corps of Engineers Bay Model

- A similar exhibit nearby is the huge U.S. Army Corps of Engineers Bay Model in Sausalito, CA
- Free to visit
- Built in 1956-57
- Digital, not mechanical tide computer
- Tides forced with pumps



Credit: US Federal Government

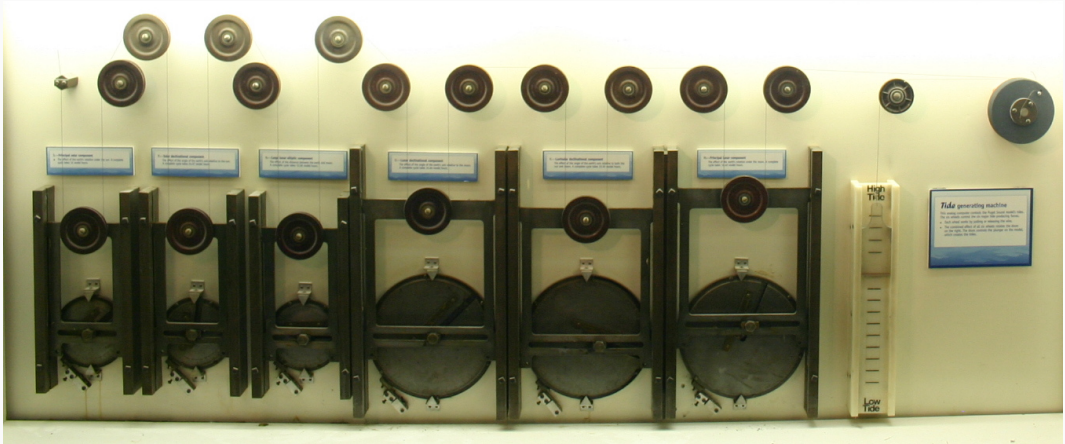
Mechanical Tide Computers

PacSci Tide Generating Machine



Credit: Photo by Ryan Somma of ideonex.com

PacSci Tide Generating Machine Detail



Credit: Photo by Ryan Somma of ideonex.com

The Mechanism

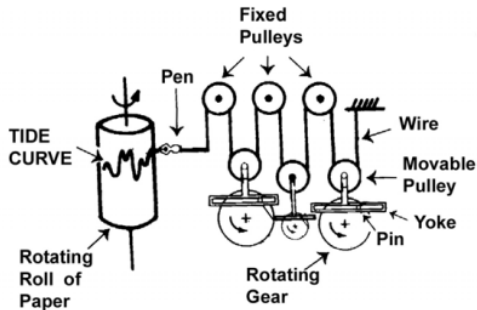


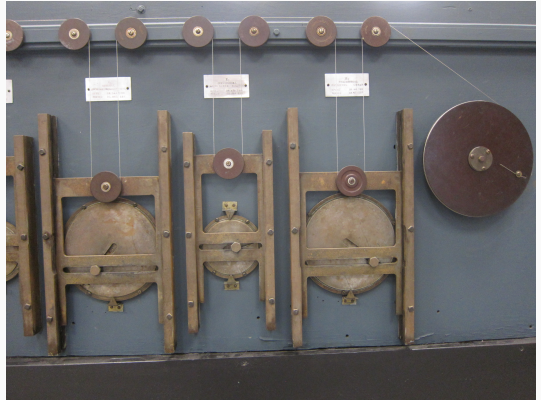
Figure 1.2. Gear and pulley system of an early analog tide predicting machine. Each gear and pulley combination represents one tidal harmonic constituent. The wire running over all the pulleys sums the motions and moves a pen on a moving roll of paper to draw the tide curve.

Credit: Bruce B. Parker, Ph.D, NOAA

UW Oceanography Tide Computer

- I visited the Puget Sound model at the University of Washington School of Oceanography to take photos and videos of the computer in action.
- A thank you to Bill Nitsche. He was very generous with his time.

Play video

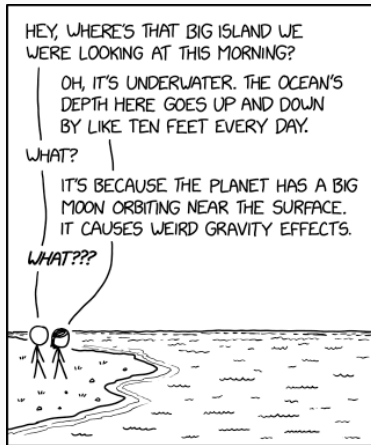


How Does This Work?

- How does such a simple machine predict the tides?

What Are the Tides?

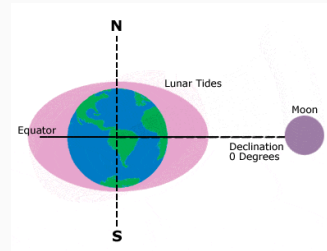
Obligatory XKCD Reference



PEOPLE HERE ARE USED TO THEM, BUT TIDES ARE ONE OF THE WEIRDEST AND MOST SCI-FI ELEMENTS OF LIFE ON EARTH.

Astronomical Tides: Definition

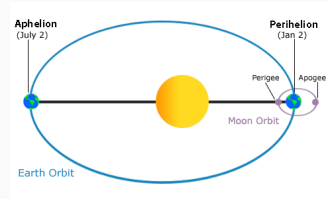
- Motion of liquid water on Earth that arises from the gravitational pull from the Moon and Sun.
- Each pull forms two bulges of water, one towards the pulling body and one away from it.
- The solid Earth spins underneath these bulges.
 - Solar day: 24 hours
 - Lunar day: 24 hours 50 minutes



Credit: NOAA

Astronomical Tides: Definition

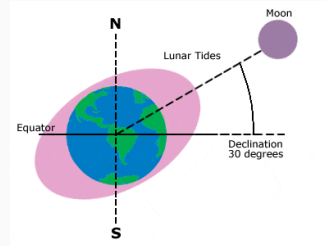
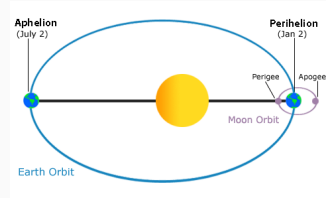
- Tides are higher when the Sun, Earth, and Moon are lined up (spring tides).
- Tides are lower when Sun, Earth, and Moon form a 90 degree angle (neap tides).
- Alternatively, beat frequencies between the solar and lunar day length create larger and smaller tides throughout the lunar orbit.



Credit: NOAA

Astronomical Tides: Orbits

- The Sun-Earth-Moon orbital system and tidal forces repeat in a 18.6 year cycle.
- The magnitude and direction of tidal forces change over time because:
 - The orbits are elliptical (changing distance)
 - The equator is tilted 23 degrees from Earth's orbital plane
 - The Moon's orbit is tilted 5 degrees from the Earth's orbital plane



Credit: NOAA

Astronomical Tides: Topography

- The topography of the sea floor and coast affects how water sloshes around.
- Topography, friction, and inertia introduce delays (phase shifts) from an ideal spherical and frictionless Earth entirely covered in water.

Astronomical Tides: Hydrology

- In effect, tides are very long waves, hundreds or thousands of miles long.
- In shallow water non-linear mixing processes give rise to:
 - Overtides: Harmonics of the astronomical frequencies
 - Compound Tides: New tidal constituents resulting from the interaction of two or more astronomical frequencies
- This should sound familiar to electrical engineers (harmonics and frequency mixing) and musicians (overtones and partials).

Meteorological Tides

- Weather has a great impact on actual tide heights.
- Atmospheric pressure can suppress or reinforce low and high tides.
- Storm surges during hurricanes are an extreme example.
- These effects are not as predictable as astronomical tides.
 - Not included in the model

- River flows also affect the tides near estuaries.
- And these are also affected by weather.
 - Not included in the model

Predicting Tides with Math

- Orbits and rotations are periodic events.
- They happen in regular, measurable, and predictable cycles.
- We have math for that!

Thanks Fourier!



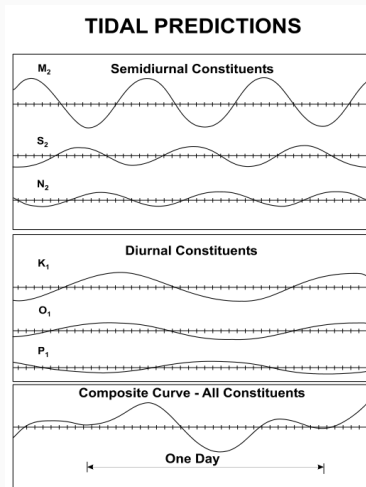
Jean-Baptiste Joseph Fourier, 1768–1830

Credit: Dr. Ele Willoughby

- It will surprise no one here that you can approximate a function using the sum of sinusoids.
- Many mathematicians studied the tides, including Isaac Newton, Pierre-Simon Laplace, and Lord Kelvin.
- Lord Kelvin was the first to make tide prediction math practical and invented the mechanical analog computer to calculate tides.

Prediction Math

- Each astronomical component of the tides at a given location can be represented with a single sinusoid wave.
- Each wave is called a "harmonic constituent" by oceanographers.
- Add up all the harmonic constituents and we have a tide predictor.
- This is like Fourier Synthesis.



Fourier Synthesis

One harmonic constituent:

$$\textit{height} = A \cos(\omega t + \varphi)$$

A Amplitude

ω Frequency

φ Phase

t Time

$$\begin{aligned} \text{tides} = & A_1 \cos(\omega_1 t + \varphi_1) \\ & + A_2 \cos(\omega_2 t + \varphi_2) \\ & + A_3 \cos(\omega_3 t + \varphi_3) \\ & + A_4 \cos(\omega_4 t + \varphi_4) \\ & + A_5 \cos(\omega_5 t + \varphi_5) \\ & + \dots \\ & + A_{37} \cos(\omega_{37} t + \varphi_{37}) \end{aligned}$$

- Where we will get the data to plug into this equation?

Harmonic Constituents

Thanks NOAA!

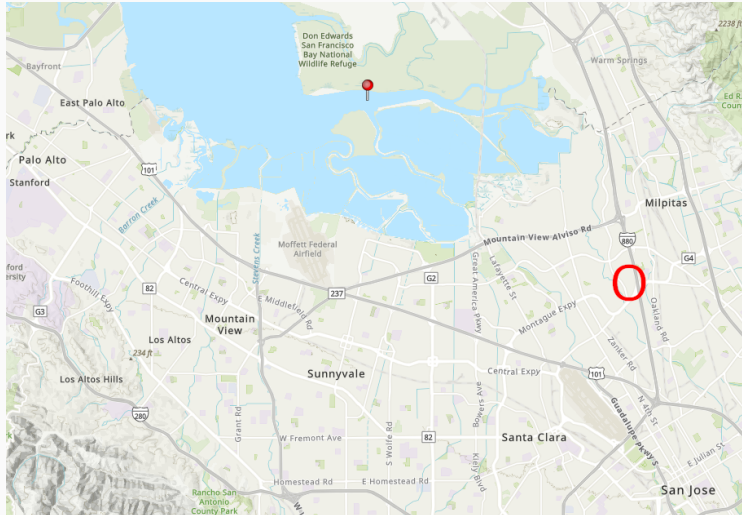
- NOAA publishes harmonic constituents for 1274 stations (as of 2018)
- Minor updates quarterly
- Major updates every 19 years
 - The National Tidal Datum Epoch (NTDE)
 - Based on 19 years of water level data
 - Captures a whole 18.6 year cycle
- The present National Tidal Datum Epoch (NTDE) is 1983 through 2001.
- An update is coming next year, based on data from 2002 through 2020.

Where to find them

- List:
 - <https://tidesandcurrents.noaa.gov/stations.html?type=Harmonic+Constituents>
- Map:
 - <https://tidesandcurrents.noaa.gov/map/index.html?type=datums>

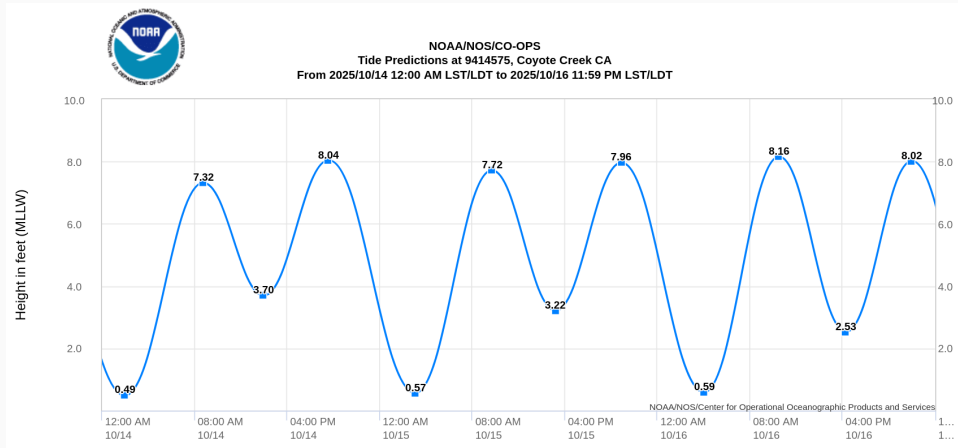
Example: Coyote Creek, CA

NOAA station 9414575 is closest to this conference.



Example: Coyote Creek, CA

- NOAA tide prediction for the station closest to this conference:



- Link, and link to nearest Water Level Station to check live results

How Are They Determined?

- Tide height is sampled hourly using acoustic water level sensors.
- The level sensors low-pass filter to remove short-term waves.
- Least squares fit of sinusoids to get harmonic constituents
 - Not a simple inverse Fourier transform or Fourier series analysis
 - The harmonic constituent frequencies are known in advance and don't fall exactly on DFT bin frequencies.
 - Least squares doesn't need regularly spaced data, so missing data is less of a problem.
- No knowledge of hydrology needed, just local measurements.

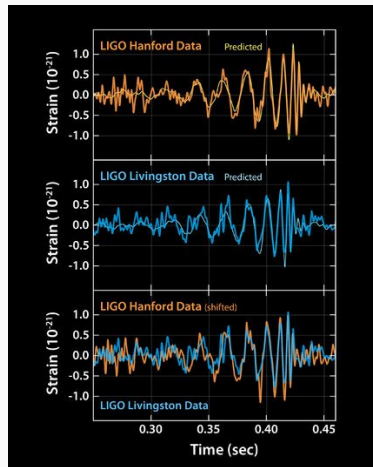
Sonification

What is Sonification?

- Sonification uses audio to present data.
- Visualization uses light to present data.
- Like sight, hearing is a highly developed sense which our brain can interpret very quickly.

Sonification Example

- The 2015 LIGO measurement of two black holes colliding. The data represents the gravitational waves that reached the Earth 1.3 billion years after the event.
- None of us have gravitational wave ears in our biology, but with a bit of software we can experience the data with our sound wave ears. (Play)
- More fun than looking at a squiggle on a graph.

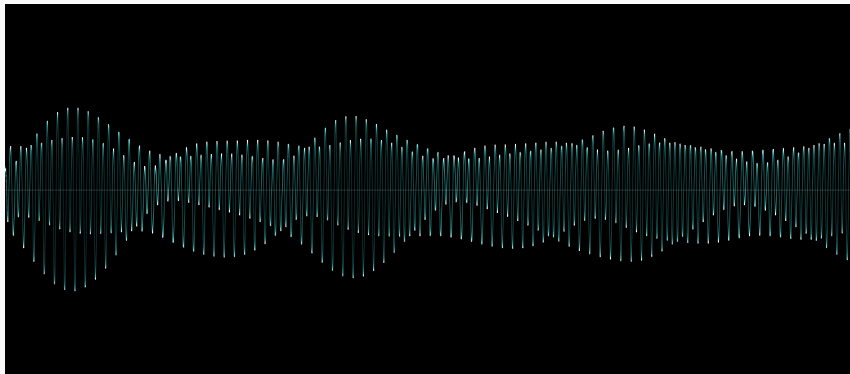


Sonification: Accessibility

- Sonification is useful too.
- Sonification makes data more accessible.
- Helps both sighted and blind people perceive new aspects of data.
- Wanda Diaz Merced gave an inspirational presentation about losing her sight in the middle of her astronomy career and using sonification to carry on.

Sonifying the Tides

Coyote Creek, CA Sonification



- Running 36 million times faster than real-time
- Six hours up, six hours down becomes 600 us up, 600 us down
- 10 seconds of audio represents 11.4 years
- Audio sample rate 44.1 kHz, 22.7 us per sample

Play audio

Sonifying All the Tides

- I posted 10 second clips for all 1274 NOAA stations on my blog.
- <https://remcycles.net/blog/tides.html>

- wav png csv amp phase 9414458 San Mateo Bridge, CA
- wav png csv amp phase 9414501 REDWOOD CREEK,C.M. NO. 8,S.F.BAY, CA
- wav png csv amp phase 9414509 Dumbarton Bridge, CA
- wav png csv amp phase 9414523 Redwood City, CA
- wav png csv amp phase 9414575 Coyote Creek, CA
- wav png csv amp phase 9414688 San Leandro Marina, CA
- wav png csv amp phase 9414746 PARK STREET BRIDGE, CA
- wav png csv amp phase 9414750 Alameda, CA
- wav png csv amp phase 9414764 Oakland Inner Harbor, CA
- wav png csv amp phase 9414767 ALAMEDA NAS, NAVY FUEL PIER, CA
- wav png csv amp phase 9414806 SAUSALITO, SAN FRANCISCO BAY, CA
- wav png csv amp phase 9414811 Bradmoor Island, CA
- wav png csv amp phase 9414816 BERKELEY,S.F.BAY, CA
- wav png csv amp phase 9414819 SAUSALITO, COE DOCK, S.F. BAY, CA
- wav png csv amp phase 9414863 Richmond, CA

- Basic idea:
 - Scale harmonic constituent frequencies to audible range
 - Generate sine waves for each constituent
 - Add them up

Generating Sine Waves

- Choose your own adventure for generating sine waves
 - Fixed-point or floating-point
 - Trig library functions (`cos()`)
 - Taylor series or other approximations
 - Table lookup
 - DDS (Direct Digital Synthesis)
 - CORDIC
 - Oscillators
 - Recurrence relations

Levine-Vicanek Quadrature Oscillator

- "A New Contender in the Quadrature Oscillator Race"
 - Rick Lyons, September 24, 2022

This blog advocates a relatively new and interesting quadrature oscillator developed by A. David Levine in 2009 and independently by Martin Vicanek in 2015.

- Generates evenly spaced sine and cosine values at a given frequency
- Only uses 3 multiplies and 3 additions per step
- Floating-point or fixed-point

Levine-Vicanek Oscillator Code

- The only state is:
 - Two constants to set the step rate (rad/sample)
 - The previous output point ($u = \cosine = I$, $v = \text{sine} = Q$)

```
typedef struct lv_osc {  
    double k1; double k2;  
    double u;  double v;  
} lv_osc_t;
```

Levine-Vicanek Oscillator Code

- u and v can be initialized to any arbitrary point on the unit circle.

```
static inline void lv_osc_init(lv_osc_t *osc,  
                               double freq_hz,  
                               double phase_rad,  
                               double sample_rate_hz) {  
    double freq_rad_per_s = 2.0*M_PI*freq_hz;  
    double theta_rad_per_sample =  
        freq_rad_per_s / sample_rate_hz;  
  
    osc->k1 = tan(theta_rad_per_sample / 2.0);  
    osc->k2 = sin(theta_rad_per_sample);  
  
    phase_rad = fmod(phase_rad, 2.0*M_PI);  
    osc->u = cos(phase_rad);  
    osc->v = sin(phase_rad);  
}
```

Levine-Vicanek Oscillator Code

```
static inline void lv_osc_step(lv_osc_t *osc) {  
    double w      = osc->u - osc->k1 * osc->v;  
    double next_v = osc->v + osc->k2 * w;  
    double next_u =      w - osc->k1 * next_v;  
    osc->v = next_v;  
    osc->u = next_u;  
}
```

- Note: You could easily extend this to do FSK or DQPSK by updating k_1 and k_2 or u and v between steps.

- A C program sonifies all 1274 NOAA stations
 - 10 seconds of 44.1 kHz mono audio for each station
 - 12740 s = 212 min = 3.54 hr of audio
 - 1.1 GB of .wav files
- 3 minutes 4 seconds using C and `stdlib cos()`
- 24 seconds using C and Levine-Vicanek oscillator
- 87% decrease in total run-time

Conclusion

Next Steps

- Make the sonification program more interactive
 - Add knobs to change the speedup factor and volume in real-time
 - Switch between stations with the push of a button
- Write a VCV Rack plugin
 - Open-source analog synthesizer simulator
 - Much easier to make music with the output or use it in other ways
 - Tune the speedup factor for each piano key (principal lunar semi-diurnal constituent, M2)
 - Amplitude or frequency modulate a tone (LFO)
- Browser implementation in Javascript
- Start a Poseidon themed synth band?

- Thank you for your time!
- Code and slides will be posted on my blog at:
 - <https://remcycles.net>.

- Now for your questions!



Thank you

signalprocessingsummit.com

Bonus Slides

NOAA has published a lot of information about how harmonic constituents are determined and used to predict tides:

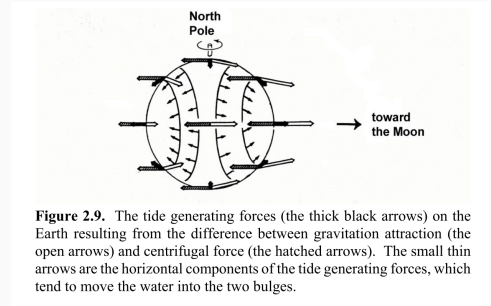
- https://tidesandcurrents.noaa.gov/about_harmonic_constituents.html
- NOAA Special Publication NOS CO-OPS 1 - Tidal Datums and Their Applications
- NOAA Special Publication NOS CO-OPS 3 - Tidal Analysis and Predictions
- NOAA Special Publication No. 98: Manual of Harmonic Analysis and Prediction of Tides

Start with easy mode here:

- <https://scijinks.gov/about/k-12-education/oceans-coasts/what-causes-tides>

Tidal Forces

- The force of gravity is inversely proportional to the square of distance.
- The near side of the Earth facing the Moon feels a stronger pull than the far side.
- This gravity gradient minus a uniform inertial force away from the Moon gives rise to two tidal "bulges", one on the near side and one on the far side.



Tidal Forces

- Tidal forces are inversely proportional to the cube of distance.
- So the nearby Moon has a greater influence on the tides than the more distant Sun.
 - The Sun's influence is about 47% of the Moon's influence.
- Tidal forces change over time due to elliptical orbits.

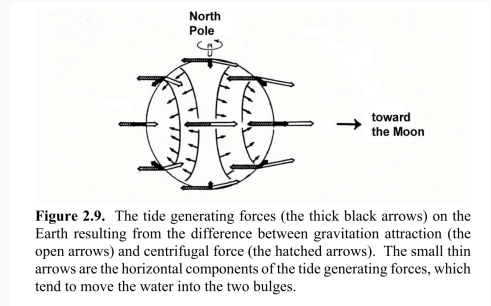
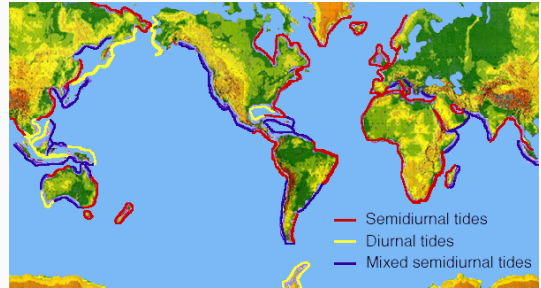
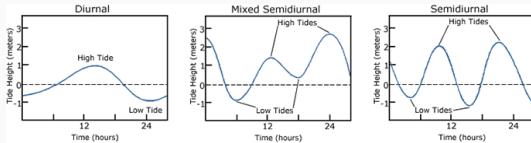


Figure 2.9. The tide generating forces (the thick black arrows) on the Earth resulting from the difference between gravitation attraction (the open arrows) and centrifugal force (the hatched arrows). The small thin arrows are the horizontal components of the tide generating forces, which tend to move the water into the two bulges.

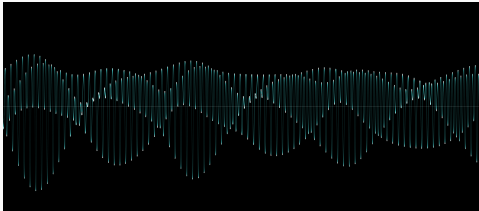
Types of Tides

- Dan Boschen previously noted a difference in sound between east coast tides and west coast tides.
- There **are** different types of daily tides.



Credit: NOAA

Seattle vs Boston



- Seattle, WA
- Mixed Semidiurnal



- Boston, MA
- Semidiurnal

Terminology

Diurnal Tide	Rise and fall once a day
Semidiurnal Tide	Rise and fall twice a day
Mixed Semidiurnal Tide	Rise and fall twice a day, with different maxima and minima

What is the Phase Relative To?

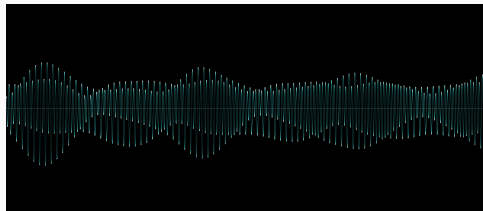
- I'm still trying to find the exact answer to this question.
- The formula for the height of one tidal constituent, h_1 , at $t = 0$, can be written as:

$$h_1 = fH\cos(V_o + u - \kappa)$$

- But the date and time for $t = 0$ is not clearly specified.
- I **think** it's the beginning of the year (plus a year specific adjustment), but have not confirmed this yet.
- The "equilibrium tide" is a very simplified model of tides assuming a perfectly spherical Earth where water responds instantly to the effect of gravity.
- The phase for a constituent represents the delay between an astronomical event's immediate effect on the equilibrium tide model and the actual effect at the measurement station.
 - For example, the lunar day, when the Moon is directly overhead.

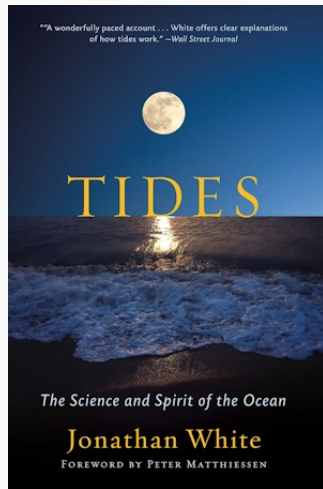
Visualizing Waveforms Nicely

- oscillo.pl Perl script by WindyTan in Finland
- Renders plots that look like an analog oscilloscope trace
- Upsamples a sound file using SoX
- Renders the upsampled image using bilinear interpolation and a pleasing gradient
- Lots of other great posts on her blog
 - Good inspiration and DSP project ideas!



Tides, by Jonathan White

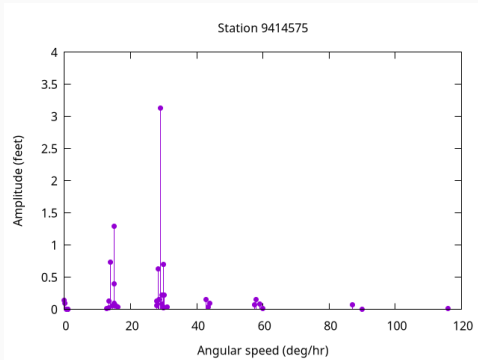
- Tides: The Science and Spirit of the Ocean, by Jonathan White
- This is a very interesting and readable book about many aspects of tides, technical and cultural.
- Jonathan wrote beautifully about mechanical tide computers, imagining the size and sound of a machine calculating 400 harmonic constituents.
- He wrote in the book that he had never seen one in person.
- He passed away in 2023, but I hope he did see one before he passed.



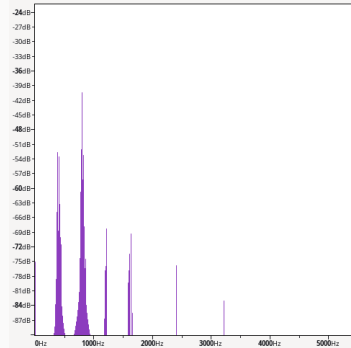
An Inventory of Tide Predicting Machines

- Lots of fun photos of historic tide machines here:
 - https://nora.nerc.ac.uk/id/eprint/513660/1/NOC_R%26C_56.pdf
- Many are viewable in museums.

Coyote Creek, CA Sonification Data



- Input:
 - Harmonic constituent amplitudes



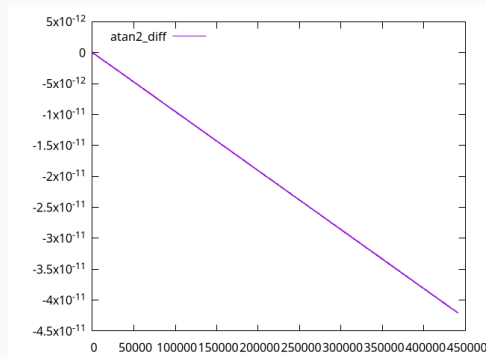
- Output:
 - FFT of audio

Table 3. Constituents of the Puget Sound model tide computer

Symbol	Function	Type	Speed ($^{\circ}$ /hr)	Period (hours)
M_2	Principal Lunar	Semidiurnal	28.9841042	12.4206012
N_2	Lunar Elliptical	Semidiurnal	28.4397296	12.6583482
K_1	Lunisolar Declinational	Diurnal	15.0410686	23.9344697
O_1	Lunar Declinational	Diurnal	13.9430356	25.8193417
P_1	Solar Declinational	Diurnal	14.9589314	24.0658902
S_2	Principal Solar	Semidiurnal	30.0000000	12.0000000

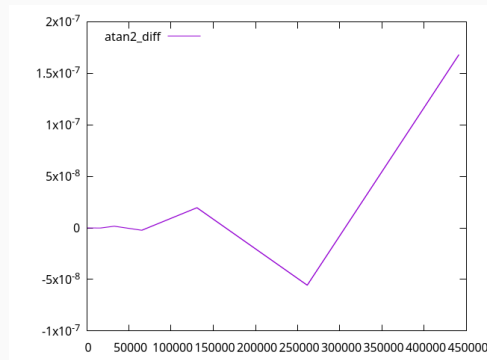
Levine-Vicanek Drift

- Phase error comparison vs phase accumulator and stdlib `cos()` and `sin()`
- Comparing phase using `atan2()` on quadrature output
- Unwrapping trig phase accumulator between 0 and 2π
- 10 seconds of 440 Hz tone in 44.1 kHz audio (441000 samples)
- $1.68\text{e-}7$ rad/s
- How long does it take to accumulate a whole cycle of error?
 - 11.86 years.



Levine-Vicanek Drift

- Phase error comparison vs phase accumulator and stdlib `cos()` and `sin()`
- **Not unwrapping** trig phase accumulator between 0 and 2π
- Errors in trig functions with large arguments quickly dwarf any accumulated phase error in the Levine-Vicanek oscillator.



Even Faster Cosines

- I stumbled upon this recurrence relation two days ago:
 - $\cos((n+1)\theta) = 2\cos(\theta)\cos(n\theta) - \cos((n-1)\theta)$
 - Real Computing Made Real, by Forman S. Acton. Page 173.
- This is the "Chebyshev method" multiple angle formula.
- Only a single multiply and subtraction is needed per step.
- Easy to initialize to any desired phase
- Two in parallel would make a quadrature oscillator
 - 2 multiplies and 2 subtractions per step

Even Faster Cosines Source

```
typedef struct recur_osc {
    double cos; double last_cos; double coeff;
} recur_osc_t;

static inline void recur_osc_init(recur_osc_t *osc,
                                  double freq_hz,
                                  double phase_rad,
                                  double sample_rate_hz) {
    double rad_per_s = 2.0*M_PI*freq_hz;
    double omega_rad_per_sample = rad_per_s / sample_rate_hz;
    osc->coeff = 2.0 * cos(omega_rad_per_sample);
#ifdef NO_UNWRAP
    phase_rad = fmod(phase_rad, 2.0*M_PI);
#endif
    osc->last_cos = cos(phase_rad - omega_rad_per_sample);
    osc->cos = cos(phase_rad);
}
```

Even Faster Cosines Source

```
static inline void recur_osc_step(recur_osc_t *osc) {  
    double next_cos = osc->coeff * osc->cos - osc->last_cos;  
    osc->last_cos = osc->cos;  
    osc->cos = next_cos;  
}
```

Even Faster Cosines Benchmark

- Just 17 seconds for whole tides program
- 91% decrease in total run-time from `stdlib cos()`
- Benchmark simply calculating $2^{32} - 1$ cosines:

Oscillator	Time (s)
<code>cos()</code>	30
lv	19
recur	8

- This presentation was made with:
Emacs, Emacs Calc, Org Mode, LaTeX, Beamer, GNU Make, GCC, Gnuplot, Awk, Bash, Tcl, Perl, GIMP, Firefox, LibreOffice, pdftk, and git.
- Thank you to the open source community for making these tools available.
- No LLM chatbots were harmed or consulted in the making of this presentation or code.



Thank you

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